

Exercícios - Cálculo 2 - 11/09/2015

Exercício 1: Resolver as integrais por substituição:

$$a) \int 7x \sqrt{5+4x^2} dx = 7 \int x(5+4x^2)^{1/2} dx =$$

$$\left| \begin{array}{l} u = 5 + 4x^2 \\ du = 8x dx \rightarrow \frac{du}{8} = x dx \end{array} \right.$$

$$= 7 \int u^{1/2} \frac{du}{8} = \frac{7}{8} \int u^{1/2} du = \frac{7}{8} \frac{u^{3/2}}{3/2}$$

$$= \frac{7}{8} \cdot \frac{2}{3} \cdot u^{3/2} = \frac{7}{12} (5+4x^2)^{3/2} + C$$

$$b) \int \sqrt{3t^2+1} dt = \int \sqrt{t^2(3t^2+1)} dt =$$

$$= \int t \cdot \sqrt{3t^2+1} dt = \int u^{1/2} \frac{du}{6} =$$

$$u = 3t^2 + 1 \rightarrow du = 6t dt = \frac{du}{6} = t dt$$

$$= \frac{1}{6} \cdot \frac{u^{3/2}}{3/2} = \frac{1}{6} \cdot \frac{2}{3} \cdot u^{3/2} = \frac{1}{9} (3t^2+1)^{3/2} + C$$

$$c) \int \frac{e^t dt}{e^{2t} + 16} = \int \frac{e^t dt}{(e^t)^2 + (4)^2} = \int \frac{du}{u^2 + (4)^2} =$$

$$u = e^t \rightarrow du = e^t dt$$

$$= \frac{1}{4} \operatorname{arctg} \frac{u}{4} = \boxed{\frac{1}{4} \operatorname{arctg} \left( \frac{e^t}{4} \right) + C}$$

Exercício 2: Resolva as integrais por partes:

$$a) \int x^2 \operatorname{sen}(5x) dx = -x^2 \frac{1}{5} \cos(5x) - \int -\frac{1}{5} \cos(5x) \cdot 2x dx =$$

$$u = x^2 \rightarrow du = 2x dx$$

$$dv = \operatorname{sen}(5x) dx \rightarrow v = \int \operatorname{sen}(5x) dx = \int \operatorname{sen} u \frac{du}{5} = -\frac{1}{5} \cos 5x$$

$$w = 5x \quad dx = \frac{dw}{5}$$

$$dw = 5 dx \rightarrow \frac{dw}{5} = dx$$

$$= -\frac{x^2 \cos(5x)}{5} + \frac{2}{5} \int x \cos(5x) dx =$$

$$= -\frac{x^2 \cos(5x)}{5} + \frac{2}{5} \left( \frac{x \operatorname{sen}(5x)}{5} - \frac{1}{5} \int \operatorname{sen}(5x) dx \right) = \dots$$

$$\int x \cos(5x) dx = \frac{x \operatorname{sen}(5x)}{5} - \frac{1}{5} \int \operatorname{sen}(5x) dx$$

$$u = x; du = dx$$

$$dv = \cos 5x dx \rightarrow v = \int \cos 5x dx = \int \cos w \frac{dw}{5} = \frac{1}{5} \operatorname{sen}(5x)$$

$$w = 5x \rightarrow \frac{dw}{5} = dx$$

$$dw = 5 dx$$

