G A B A R I T O

Lista de Exercícios sobre integrais indefinidas – Atividade Extra-curricular

Atividade para reposição da aula de 08/09/2018

Entregar em 15/09/2018

Resolva as integrais imediatas abaixo:

a) $\int (\frac{5x^4+8x^3-3x^2-5}{x^3})dx$

$$\int (\frac{5x^4+8x^3-3x^2-5}{x^3})dx = \int \frac{5x^4}{x^3}dx+\int \frac{8x^3}{x^3}dx-\int \frac{3x^2}{x^3}dx-\int \frac{5}{x^3}dx =$$

$$= 5\int x\,dx+8\int dx-3\int \frac{1}{x}dx-5\int \frac{1}{x^3}dx = \frac{5x^2}{2} + 8x - 3\ln|x| + \frac{5}{2x^2} + c$$

b) $\int 4x^3\cdot\sqrt[4]{x^3}\,dx$

$$\int 4x^3\cdot\sqrt[4]{x^3}\,dx = 4\int x^3\cdot x^{\frac{3}{4}}dx = 4\int x^{\frac{15}{4}}dx = 4\frac{x^{\frac{19}{4}}}{\frac{19}{4}} = \frac{16}{19}x^{\frac{19}{4}} + c$$

c) $\int (\frac{2x^3}{2x^3+5})dx$

Este item foi mal elaborado para resolver como integral imediata

d) $\int \sqrt{\frac{4}{x^4-x^2}}dx$

$$\int \sqrt{\frac{4}{x^4-x^2}}dx \;=\; \int \frac{\sqrt{2^2}}{\sqrt{x^2(x^2-1)}}dx \;=\; \int \frac{2}{x\sqrt{x^2-1}}dx \;=\; 2\int \frac{dx}{x\sqrt{x^2-1}} \;=$$
$$= \; 2\left[\frac{1}{1}arcsec\,\frac{x}{1}\right] \;=\; 2\,arcsec\,x \;+\; c$$

e) $\int (\frac{e^t}{2}+\sqrt{t}+\frac{1}{t})dt$

$$\int (\frac{e^t}{2}+\sqrt{t}+\frac{1}{t})dt \;=\; \frac{1}{2}\int e^t dt \;+\; \int t^{\frac{1}{2}}dt \;+\; \int \frac{dt}{t} \;=\; \frac{1}{2}e^t \;+\; \frac{2}{3}t^{\frac{3}{2}} \;+\; \ln|t| \;+\; c$$

Determine uma primitiva F da função $f(x)=x^{(\frac{2}{3})}+x$ que satisfaça a identidade F(1) = 1.

$$F(x)=\int x^{\frac{2}{3}}dx+\int x\,dx=\frac{3x^{\frac{5}{3}}}{5} \;+\; \frac{x^2}{2} \;+\; c$$

Mas como F(1) = 1 , então temos:

$$1=\frac{3(1)^{\frac{5}{3}}}{5} \;+\; \frac{(1)^2}{2} \;+\; c$$
$$1=\frac{3}{5} \;+\; \frac{1}{2} \;+\; c \;\;\rightarrow\;\; c=\frac{-1}{10}$$

Portanto : $F(x)=\frac{3x^{\frac{5}{3}}}{5} \;+\; \frac{x^2}{2} \;-\; \frac{1}{10}$

Calcule as integrais indefinidas abaixo, usando o método da substituição:

a) $\int (x^3-2)^{(\frac{1}{7})}\cdot x^2 dx$

Fazendo $u=(x^3-2) \;\rightarrow\; du=3x^2dx \;\rightarrow\; \frac{du}{3}=x^{2dx}$. Assim:

$$\int (x^3-2)^{(\frac{1}{7})}\cdot x^2dx \;=\; \int u^{\frac{1}{7}}\frac{du}{3} \;=\; \frac{1}{3}\int u^{\frac{1}{7}}du \;=\; \frac{1}{3}\frac{u^{\frac{8}{7}}}{\frac{8}{7}} \;=\; (\frac{1}{3})(\frac{7}{8})(x^3-2)^{\frac{8}{7}} \;=\; (\frac{7}{24})(x^3-2)^{\frac{8}{7}} \;+\; c$$

b) $\int 5x\cdot\sqrt{4-3x^2}\,dx$

Fazendo $u = 4-3x^2 \rightarrow \frac{du}{-6} = x\,dx$. Assim:

$$\int 5x\cdot\sqrt{4-3x^2}\,dx = 5\int\sqrt{u}\,\frac{du}{-6} = -\frac{5}{6}\int u^{\frac{1}{2}}du = -\frac{5}{6}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} =$$

$$(\frac{-5}{6})(\frac{2}{3})(4-3x^2)^{\frac{3}{2}} = (\frac{-5}{9})(4-3x^2)^{\frac{3}{2}}+c$$

c) $\int (\frac{x}{\sqrt[5]{x^2-1}})dx$

Fazendo $u = x^2-1 \rightarrow \frac{du}{2} = x\,dx$. Assim:

$$\int (\frac{x}{\sqrt[5]{x^2-1}})dx = \int \frac{1}{\sqrt[5]{u}}\frac{du}{2} = \frac{1}{2}\int u^{-(\frac{1}{5})}du = \frac{1}{2}\frac{u^{\frac{4}{5}}}{\frac{4}{5}} = \frac{1}{2}\ \frac{5}{4}\ (x^2-1)^{\frac{4}{5}} = \frac{5}{8}\ (x^2-1)^{\frac{4}{5}}+c$$

d) $\int (\frac{e^t}{[e^t+4]})dt$

Fazendo $u = e^t+4 \rightarrow du = e^t dt$. Assim:

$$\int (\frac{e^t}{[e^t+4]})dt = \int \frac{du}{u} = \ln|u| = \ln|e^t+4|+c$$

e) $\int \sqrt{x^2+2x^4}\,dx$

Primeiro fazemos

$$\int \sqrt{x^2+2x^4}\,dx = \int \sqrt{x^2(1+2x^2)}\,dx = \int \sqrt{x^2}\sqrt{(1+2x^2)}\,dx = \int x\sqrt{(1+2x^2)}\,dx$$

Fazendo $u = 1+2x^2 \rightarrow \frac{du}{4} = xdx$. Assim:

$$\int x\sqrt{(1+2x^2)}\,dx = \int \sqrt{u}\frac{du}{4} = \frac{1}{4}\int u^{\frac{1}{2}}du = \frac{1}{4}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{4}\frac{2}{3}(1+2x^2)^{\frac{3}{2}} = \frac{1}{6}(1+2x^2)^{\frac{3}{2}}+c$$

Portanto: $\int \sqrt{x^2+2x^4}\,dx = \frac{1}{6}(1+2x^2)^{\frac{3}{2}}+c$