Atividade 1 - Cálculo 2 - Fabricação Mecânica

Gabarito

1) $\int x^3\sqrt{x}\,dx = \int x^3 \cdot x^{\frac{1}{2}}\,dx = \int x^{\frac{7}{2}}\,dx =$

$= \dfrac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} = \dfrac{x^{\frac{9}{2}}}{\frac{9}{2}} = \boxed{\dfrac{2}{9}x^{\frac{9}{2}} + C}$

2) $\int \sqrt{\dfrac{4}{x^4-x^2}}\,dx = \int \dfrac{\sqrt{4}}{\sqrt{x^2(x^2-1)}}\,dx = \int \dfrac{2}{x\sqrt{x^2-1}}\,dx =$

$= 2\int \dfrac{dx}{x\sqrt{x^2-1}} = 2\,\text{arc sen}\,x + C$

3) $\int \dfrac{-9x^3+6x^2+1}{x^2}\,dx = \int \dfrac{-9x^3}{x^2}\,dx + \int \dfrac{6x^2}{x^2}\,dx + \int \dfrac{1}{x^2}\,dx =$

$= \int -9x\,dx + \int 6\,dx + \int \dfrac{1}{x^2}\,dx = \dfrac{-9x^2}{2} + 6x + \left(-\dfrac{1}{x}\right) =$

$\dfrac{-9x^2}{2} + 6x - \dfrac{1}{x} + C$

4) $\int \dfrac{t^2}{\sqrt[3]{1+t^3}}\,dt = \int \dfrac{\frac{du}{3}}{\sqrt[3]{u}} = \dfrac{1}{3}\int \dfrac{du}{u^{1/3}} =$

$u = 1+t^3$; $du = 3t^2\,dt$

$\dfrac{du}{3} = t^2\,dt$

$= \dfrac{1}{3}\int u^{-\frac{1}{3}}\,du = \dfrac{1}{3}\cdot\dfrac{u^{\frac{2}{3}}}{\frac{2}{3}} = \dfrac{1}{\cancel{3}}\cdot\dfrac{\cancel{3}}{2}\cdot u^{\frac{2}{3}} = \dfrac{1}{2}\sqrt[3]{u^2}$

$= \dfrac{1}{2}\sqrt[3]{(1+t^3)^2} + C$

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5) $\int \frac{\cos x}{sen^2 x}\,dx = \int \frac{du}{u^2} = \int u^{-2}\,du = -u^{-1}$

$u = sen x$; $du = \cos x\,dx$

$= \frac{-1}{u^1} = \frac{-1}{sen x} + C$

6) $\int t\, sen(2t)\,dt = -\frac{1}{2} t \cos(2t) - \int -\frac{1}{2}\cos(2t)\,dt =$

$u = t$; $du = dt$

$dv = sen\,2t\,dt$; $v = \int sen\,2t\,dt = \int sen\,w \frac{dw}{2} = -\frac{1}{2}\cos(2t)$

$w = 2t$; $dw = 2dt$

$\frac{dw}{2} = dt$

$= -\frac{1}{2} t\cos(2t) + \frac{1}{2}\int \cos(2t)\,dt =$

$w = 2t$; $dw = 2dt \rightarrow \frac{dw}{2} = dt$

$= -\frac{1}{2} t\cos(2t) + \frac{1}{2}\int \cos(w)\frac{dw}{2} =$

$= -\frac{1}{2} t\cos(2t) + \frac{1}{4}\int \cos(w)\,dw = -\frac{1}{2} t\cos(2t) + \frac{1}{4} sen(w) =$

$= -\frac{1}{2} t\cos(2t) + \frac{1}{4} sen(2t) + C$

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7) $\int e^{2\theta} \cdot sen(3\theta)\, d\theta = e^{2\theta}\left(-\frac{1}{3}\cos(3\theta)\right) - \int -\frac{1}{3}\cos(3\theta)\, 2e^{2\theta} d\theta =$

$u = e^{2\theta};\ du = [e^{u}]' = 2e^{2\theta} d\theta$

$w = 2\theta;\ w' = 2$

$dv = sen(3\theta)\, d\theta;\ v = \int sen(3\theta)\, d\theta = \int senw \frac{dw}{3} = -\frac{1}{3}\cos(3\theta)$

$w = 3\theta;\ dw = 3d\theta$

$\frac{dw}{3} = d\theta$

$= -\frac{e^{2\theta} \cdot \cos(3\theta)}{3} + \frac{2}{3}\int e^{2\theta}\cos(3\theta)\, d\theta =$

$\int e^{2\theta} \cdot \cos(3\theta)\, d\theta = \frac{e^{2\theta} \cdot sen(3\theta)}{3} - \int \frac{1}{3} sen(3\theta) \cdot 2 \cdot e^{2\theta} d\theta$

$u = e^{2\theta};\ du = 2e^{2\theta} d\theta$

$dv = \cos(3\theta)\, d\theta;\ v = \int \cos(3\theta)\, d\theta = \frac{1}{3} sen(3\theta)\, d\theta$

$= \frac{-e^{2\theta} \cdot \cos(3\theta)}{3} + \frac{2}{3}\left[\frac{e^{2\theta} \cdot sen(3\theta)}{3} - \frac{2}{3}\right] e^{2\theta} \cdot sen(3\theta) d\theta$.

Portanto:

$\int e^{2\theta} \cdot sen(3\theta)\, d\theta = -\frac{e^{2\theta} \cdot \cos(3\theta)}{3} + \frac{2e^{2\theta} \cdot sen(3\theta)}{9} - \frac{4}{9}\int e^{2\theta} \cdot sen(3\theta)\, d\theta$

$\frac{13}{9}\int e^{2\theta} \cdot sen(3\theta)\, d\theta = -\frac{e^{2\theta}\cos(3\theta)}{3} + \frac{2e^{2\theta} sen(3\theta)}{9}$

$\int e^{2\theta} \cdot sen(3\theta)\, d\theta = \frac{\cancel{9}^{3}}{13} \cdot \frac{-1}{\cancel{3}} \cdot e^{2\theta}\cos(3\theta) + \frac{\cancel{9}}{13} \cdot \frac{2}{\cancel{9}} \cdot e^{2\theta} sen(3\theta)$

$\int e^{2\theta} sen(3\theta)\, d\theta = \frac{3}{13} e^{2\theta}\cos(3\theta) + \frac{2}{13} e^{2\theta} sen(3\theta) + C$

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8) $\int_{-1}^{1} \frac{x^2}{\sqrt{x^3+9}}\,dx = \int_{-1}^{1} \frac{\frac{du}{3}}{u^{1/2}} = \frac{1}{3}\int_{-1}^{1} u^{-1/2}\,du =$

$u = x^3+9$; $du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \frac{1}{3}\left[\frac{u^{1/2}}{\frac{1}{2}}\Big|_{-1}^{1}\right] = \frac{1}{3}\left[2\cdot\sqrt{x^3+9}\Big|_{-1}^{1}\right] =$

$= \frac{2}{3}\left[\left(\sqrt{1+9}\right) - \left(\sqrt{-1+9}\right)\right] = \frac{2}{3}\left[\sqrt{10} - \sqrt{8}\right]$

9) $\int_{0}^{1} x^2(1+2x^3)^5\,dx = \int_{0}^{1} u^5 \frac{du}{6} = \frac{1}{6}\int_{0}^{1} u^5\,du =$

$u = 1+2x^3$; $du = 6x^2\,dx \rightarrow \frac{du}{6} = x^2\,dx$

$= \frac{1}{6}\left[\frac{u^6}{6}\Big|_{0}^{1}\right] = \frac{1}{36}\left[(1+2x^3)^6\Big|_{0}^{1}\right] = \frac{1}{36}\left[\left(3^6\right) - \left(1\right)\right] = 20{,}222$

10) Calcule a área da região limitada pelo gráfico da função $y = \sqrt{1+x}$, pelo eixo x e $x=0$ e $x=3$.

$A = \int_{0}^{3} \sqrt{1+x}\,dx = \int u^{1/2}\,du = \left[\frac{(1+x)^{3/2}}{3/2}\Big|_{0}^{3}\right] =$

$u = 1+x$; $du = dx$

$= \frac{2}{3}\left[\sqrt{(1+x)^3}\Big|_{0}^{3}\right] = \frac{2}{3}\left[\left(\sqrt{4^3}\right) - \left(\sqrt{1^3}\right)\right] =$

$= \frac{2}{3}\left[8 - 1\right] = \frac{2}{3}\left[7\right] = \frac{14}{3}$ uA